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The effect of changing climate on the frequency of absolute extreme events

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1 Introduction

In some areas of climate impact analysis, the possible impact of a changing mean climate has been dismissed by some writers either because of a belief that society can adapt to a slowly changing mean and/or because expected rates of future changes lie within or not far outside those experienced in the past. The two standard counter arguments to this optimistic view are: (1) the future will lead to much longer periods of protracted change in one direction, with final conditions well into the no-analogue region; and/or (2) the main impacts will accrue through changes in the frequency of extremes. In the literature on greenhouse effect, lip service is often paid to the effect of changes in the frequency of extremes. But just how will a slowly changing mean affect the frequency of extremes? Quantitative discussions of this subject are rare,

Editor's Note: Occasionally papers are published in less limited distribution journals that deserve to be more widely read. The following paper by Tom Wigley is one of these – it presents some simple analyses of extreme event theory that lie on the boundary between textbook and scientific journal material and so tend to slip between the cracks. Since the original publication of this paper, interest in and understanding of extreme events has developed rapidly. To provide a bridge to the present state of the art, an updated review of the subject is given in the accompanying paper by Dan Cooley.

Author's Note: I wrote this paper more than 20 years ago for publication in the Climatic Research Unit's in-house journal "Climate Monitor". This was at a time when the interest in possible anthropogenic changes in the frequency of extreme events was just beginning, and the paper was presented primarily as a pedagogical piece to explain certain relatively simple aspects of extreme-event theory such as risk and return period. In addition, I made some predictions of the likelihood of future extremes that have been borne out by, for example, the record-breaking hot summer of 2003 in Europe. These simple results have not appeared in the literature subsequently. The original paper was not refereed, but this reprint was refereed for publication in Climatic Change.

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and, surprisingly, there are some extremely simple analyses that have not yet been carried out. The purpose of this note is to remedy this deficiency.

2 The constant climate case

One of the underlying assumptions of classic extreme event theory is that the events are sampled from a stationary population - i.e. a population whose statistical properties (mean, variance, skewness, etc.) do not vary with time. For a future greenhouse-influenced world, this is clearly an invalid assumption. Nevertheless, I will begin with this assumption, both as a base case and in order to define some terminology. All of the results in this Section are well known.

Consider an extreme event for a variable X defined by

$$X > X_c$$

where X_c is some critical value. X could be the maximum summer temperature, the mean soil moisture deficit over the growing season, storm surge or river flood height, etc. In the present analysis, I will assume that successive X values are uncorrelated. This is true for a large number of impact-related variables; but there are, equally, a number of variables for which it is not true. If the distribution function for X is f(X) then the probability of a single extreme occurring during some specified sampling interval is given by

$$\Pr\{X > X_c\} = p = \int_{X_c}^{\infty} f(X)dx \tag{1}$$

(In the following, the sampling interval will be taken as one year.) The probability, p, is the inverse of the <u>return period</u>, $\tau(p)$, which is defined as the expected waiting time between successive occurrences of extremes (see below). For example, if $\tau(p) = 100$ years, this means that, on average, we would have to wait 100 years after an occurrence before the event occurred again.

Let us suppose that n is the actual number of years to the first occurrence of the designated extreme event - i.e. the extreme first occurs in year n. The probability that this will happen is simply

$$P_n = (1-p)^{n-1}\,p \tag{2}$$

Note that the <u>mode</u> for the distribution of n - i.e. the individual year in which the first occurrence is most likely to occur - is the first year, $n = 1$. If p is small, however, the probability P_n decreases only very slowly as n increases, so that, for quite a number of years, the probability that the first occurrence of the extreme will occur in a given year is roughly constant (and very small).

The return period is defined by

$$\tau(p) \equiv E\{n\} = \sum_{n=1}^{\infty} n\,p_n \tag{3}$$

where E{ } denotes expected value. This series can be summed to give

$$\tau(p) = 1/p \tag{4}$$

Thus, an event which occurs with probability 0.01 has a 100-year return period. Since the distribution of n is so flat, the variance of n is large. This is given by

$$\mathrm{Var}\{n\} \equiv E\left\{(n-\tau)^2\right\} = \sum_{n=1}^{\infty}(n-\tau)^2 P_n = (1-p)/(p^2) \tag{5}$$

which, for small p, is

$$\mathrm{Var}\{n\} \approx \tau^2 \tag{6}$$

Another consequence of the flatness of the distribution of n is that there is a substantial probability that the first occurrence of the extreme will occur well before the return period. This effect is encapsulated in the concept of "risk". Risk, R, is defined as the <u>cumulative</u> probability of occurrence of an extreme, i.e. the probability that a specified extreme will have occurred at least once by time t. This is given by

$$R = \sum_{n=1}^{\infty} p_n \tag{7}$$

R may be expressed either as a fraction ($0 < R < 1$) or a percentage. From Eq. 2 it follows that, provided p is constant,

$$R = 1-(1-p)^t \tag{8}$$

In the hydrological literature risk is referred to as "risk of failure", and the time t of interest is usually the design life of a structure.

As an example, consider an event with a return period of 100 years. The risk for t=1 is 1% (i.e. R=0.01). For t=5, the risk is becoming appreciable, about 5%. For t=69, the risk is 50%; for t=100 (i.e. the return period), the risk is 63%; for t=300, the risk is 95%; and so on.

In hydrology and in protecting coastal structures against storm surges, it is common procedure to estimate the extreme which corresponds to a specified risk and design life. This is equivalent to calculating the return period for given R and t. Suppose we were willing to accept a risk of 10% at t=100 years. The required value of τ may be obtained directly by inverting Eq. 8

$$\tau = \left\{1-(1-R)^{1/t}\right\}^{-1} \tag{8a}$$

which, for t=100 and R=0.1 gives $\tau = 950$ years. If τ (or its equivalent, p) is known, then the corresponding extreme can be obtained from the distribution function.

Equation 8a is well approximated by

$$\tau \approx -t/\ell n(1-R) \tag{8b}$$

which leads to the small-R result

$$\tau \approx t/R \tag{8c}$$

An even better approximation is

$$\tau \approx t/R - t/2 \tag{8d}$$

which has a maximum error of only a few years for all practical values of R and t. For example: if $R = 0.1$ and $t = 100$ years, Eq. 8d gives $\tau = 950$ years compared with the correct value of 949.6 years.

The flatness of the distribution of n makes it difficult to estimate reliably what the true return period of any given event is. If an unprecedented event occurs after, say, 100 years of observations, does this mean that the return period for the event is about 100 years, or that the event has a much longer return period, but just happened to occur, by chance, much sooner? Clearly, not much can be said on the basis of a single event, and, in general, other extremes in the record must be considered in trying to build up a picture of the likelihood of any given, well-defined extremes. There are basically two approaches possible. Which one is appropriate depends on the type of data.

The first is to consider all the data for a particular variable. For example, if one is interested in extremes of monthly-mean temperature, then all monthly means (for a particular month) can be used to define a distribution function, and the probability of $T > T_c$ (for some critical T value, T_c) is as given by Eq. 1. Alternatively, one might consider only the extremes, either all extremes above a specified threshold or only the maximum value in each year, and fit an appropriate distribution to these. An example here is the maximum storm surge (i.e. tide gauge level) in a year. One can consider just the annual maxima rather than try to fit a distribution to all tide gauge data. Whatever the method, even if the return period is known with high accuracy, the probability that the specified extreme will occur much sooner (or much later) than the return period is far from small. The risk concept allows one to account for this problem to some degree.

3 Extremes in a changing climate

What happens now if the climate is changing? Obviously, the above analyses no longer hold. If the climate changes, the probability of the event occurring once (i.e. in a single trial), p, is no longer a constant. In the situations this paper is concerned with, p will increase with time, and this will in turn reduce return periods and increase the risk of an extreme event occurring before a specified time. Just how rapidly these changes will occur depends on the rate of change of the mean of the climate variable being considered (X), and on whether there are changes in the other statistical parameters which determine the distribution of X (for example, the variance). Even if only the mean, μ changes, the effect of $d\mu/dt \neq 0$ on p will depend on the shape of the distribution function of X, f(X).

In the climate literature, very few people have considered how frequencies of extremes may change in a changing climate: examples are Mearns et al. (1984), Wigley (1985) and Hansen et al. (1988), and the work of Parry (e.g. 1978) and Parry and Carter (1985). However, in considering coastal defence systems, changes in sea level have been considered in a number of studies. Two approaches have been used. The first is simply to project past sea level trends to some future date, and use this as the base level for return period and risk calculations. The second is to assume a trend and use the method of stochastic simulation to estimate how such a trend may affect the frequency of extremes. This is the method I will use here. Since the results depend on f(X), I will have to make an assumption regarding the form of f: I will suppose that X is Normally distributed with mean μ and variance σ^2.

There is one aspect of this choice that is crucial to the analysis presented below. By assuming that all the data used in the analysis can be represented by a single distribution function, it is possible to calculate how p varies as the parameters of the distribution change. In many applications, this may not be a valid assumption since the extremes may come from one or more distinct distributions. For example, the particular weather situations that lead to extremes of temperature, precipitation, etc., may be unrelated to those which lead to the usual values of these variables. If this were the case, there could be a substantial change in the mean with no change in the frequency of the situations that produce extremes. The rest of this discussion is predicated on the assumption that average and extreme conditions are coupled.

In the stationary case, many useful results can be obtained analytically. In the "climate change" case, the equivalent results can easily be obtained by stochastic simulation. The case I will consider is one in which the mean of X increases linearly with time - corresponding to a steady warming or rise of sea level, for example. I will suppose that "a" is the rate of increase in μ using standard deviation (σ) units, i.e.

$$a = \frac{1}{\sigma}\frac{d}{d}\frac{\mu}{n}$$

where n is the number of years, so that

$$\mu = \mu_0 + a(n-1)^{\sigma}$$

where μ_0 is the initial mean.

The use of σ units means that the rate of change of μ is specified in a way which is independent of the units used to measure X, thus allowing different variables to be more easily compared. The change in p between time n=1 and time "n" depends only on the total change in μ expressed in σ units and is independent of the linear

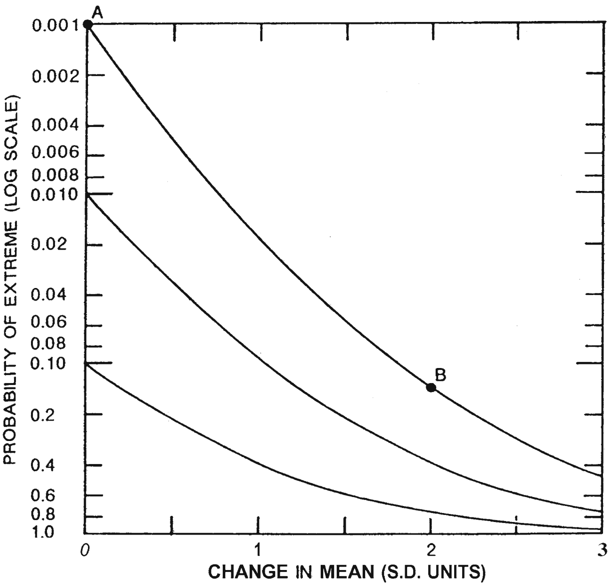

Fig. 1 Change in the probability of exceeding a specified extreme value as the mean is changed, for Normally distributed data. The three curves correspond to "initial" probabilities (i.e. prior to a change in the mean) of 0.1, 0.01 and 0.001. As an example, an event with an initial probability of p=0.001 (point A) becomes 140 times more likely (p=0.14, point B) if the mean is increased by two standard deviations

rate of change assumption. However, the effects of a change in the mean on return period and risk do depend on the linearity assumption.

Changes in p for differing initial p values are shown in Fig. 1. Obviously, as $(\mu - \mu_0)/\sigma$ becomes increasingly large, p tends to 1. The link between p and $(\mu - \mu_0)/\sigma$ is clearly nonlinear, and is determined by the assumed distribution of X.

Although Fig. 1 shows how p increases as the mean changes in the most general case, it is perhaps more informative to consider some more specific examples. I will suppose that the initial (n=1) value of p (= p_o) is known, and pose two questions :

(1) How much is the return period reduced by a trend in the mean?
(2) How much is the risk at a certain time increased by a trend in the mean?

4 Changes in the return period

The return period corresponds to the point in time at which the expected number of "successes" (i.e. extremes) is 1. In the absence of a trend, the expected number of extremes per decade is constant. With a trend, the expected number of extremes per decade increases rapidly, so one would expect a sharp decline in the return period. This is just one aspect of a noticeable shift towards smaller values of the entire distribution of first occurrence times. One can therefore ask the general question, how is the distribution of the time of first occurrence of the extreme event, $X > X_c$ affected by a trend in μ?

The shift in the distribution can be found by carrying out a large number of simulations as follows. For time n=1, using a suitable random number generator, a random value of X is selected from the base-case Normal distribution, $ND(\mu_0, \sigma)$. Then, for time n=2, a random value of X is selected from $ND(\mu = \mu_0{+}a, \sigma)$ for time n=3, a random value is selected from $ND(\mu = \mu_0{+}2a, \sigma)$ and so on until the value of X first exceeds the constant critical value X_c which defines the extreme event. This gives one realisation of the first occurrence time. The distribution of first occurrence times is obtained by repeating the procedure a few thousand times. The resulting distribution will describe how P_n, the probability that the first extreme occurs in

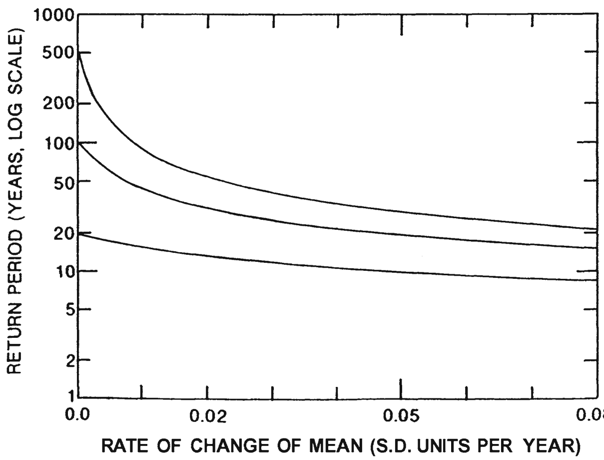

Fig. 2 Changes in the return period for an extreme event due to a trend in the mean. The three curves correspond to "no trend" return periods of 20, 100 and 500 years

year n, varies with n. It will be a generalization of Eq. 2, one which accounts for the dependence of p on n. The return period can then be calculated using Eq. 3, and the risk at any time found using Eq. 7.

The results depend on the chosen value of a, and on the critical level which defines the extreme, i.e. the X_c value. For convenience, I have considered a range of values of P_0 to define the X_c values. If a=0, then the stochastic simulation simply generates the results already given for the stationary case. Results for changes in the return period for various values of a are shown in Fig. 2. Note that the most rapid decline in return period as a increases is for small a (i.e. small values of the abscissa) and large values of the "no trend" return period (i.e large values of the intercept on the ordinate). Thus, even a small trend in the mean may markedly reduce the return period.

5 Changes in risk

It is a little more difficult to consider changes in risk in a general sense, because risk involves two parameters, the risk level itself and the future time to which that risk level applies. In the constant-climate case, this is not a problem because of the simple relationship between risk, time interval and implied return period (Eqs. 8 and 8a–d). This relationship clearly doesn't apply in the changing-climate case; in fact, in this case there is nothing to be gained by even trying to interpret a risk/time interval pair in terms of an equivalent return period. The risk and return period concepts are best considered simply as two alternative ways of assessing changes in the frequencies of extremes.

To illustrate how a changing mean affects risk, I will consider a specific time interval, 100 years, and various values of the initial risk, R_0. Changes in risk are easily calculated as a function of the trend in the mean using the stochastic simulation method. The results are shown in Fig. 3. These results are quite striking, since they indicate that risk is extremely sensitive to a change in the mean. For example, with a trend in μ of only 0.02 σ - units per year, an event with a "no trend" risk of 0.01

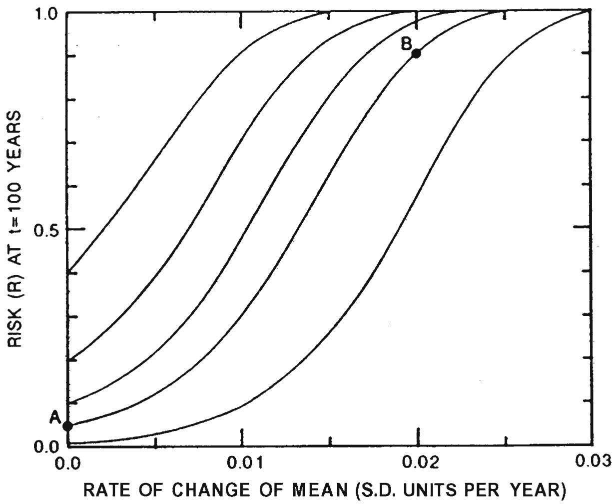

Fig. 3 Changes in the risk of occurrence of an extreme event within years due to a trend in the mean. The curves shown are for "no trend" risks of 0.01, 0.05, 0.1, 0.2 and 0.4. For a design life of t=100 years, the corresponding return periods (using Eq. 8a) are 9950, 1950, 950, 449 and 196 years. As an example, if an event has a 10% risk (R=0.1) of occurrence in 100 years in the absence of a trend (point A), then the risk would be 90% if the mean were increasing at 0.02 standard deviations per year (point B)

actually has a risk exceeding 0.5, while an event with a "no trend" risk of 0.2 has a risk which is effectively 1.

6 A specific example

To give some idea of what values of a may be realistic for future changes in climate under the greenhouse effect, I will consider monthly-mean temperatures for the summer months in England.

From observed values of Manley's Central England Temperature record (Manley 1974; Jones 1987), the means and standard deviations over the period 1881–1987 are: June, $\mu = 14.22°C$, $\sigma = 1.01°C$; July, $\mu = 15.98°C$, $\sigma = 1.17°C$; August, $\mu = 15.59°C$, $\sigma = 1.15°C$. One of the warmest summers on record is that of 1976 with temperatures of: June, 17.0°C; July, 18.7°C; August, 17.6°C. If the data were Normally distributed with means and standard deviations as given above, then these 1976 values would have probabilities of exceedance of : June, p=0.0030; July, p=0.0100; August, p=0.0402. Return periods are the inverses of these p values. For June, 1976 is the warmest in the entire record. For July and August, the warmest values on record are: July, 1983, T=19.6°C, p=0.0010; August, 1975, T=18.7°C, p=0.0034.

What rates of change can be expected in the future? These can be estimated from the results of equilibrium GCM simulations for a 2 × CO_2 world. These suggest that the mean summer warming for England is just over 4°C ranging between about 3°C and 6°C depending on the model. (I have used data from the NCAR, GFDL, GISS and OSU models, as summarized by Schlesinger and Mitchell, 1987.) Since these particular models give an average equilibrium global-mean warming of around for 4°C for 2 × CO_2, the summer warming for England can be assumed to be roughly equal to the global-mean warming value. However, the global-mean warming given by these models is at the high end of the accepted range (viz. 1.5–4.5°C) of possible values for this "climate sensitivity" parameter. If we allow for this basic model uncertainty, the range of possible 2 × CO_2 equilibrium summer warming values for England should also be about 1.5–4.5°C. (Because of the additional uncertainties

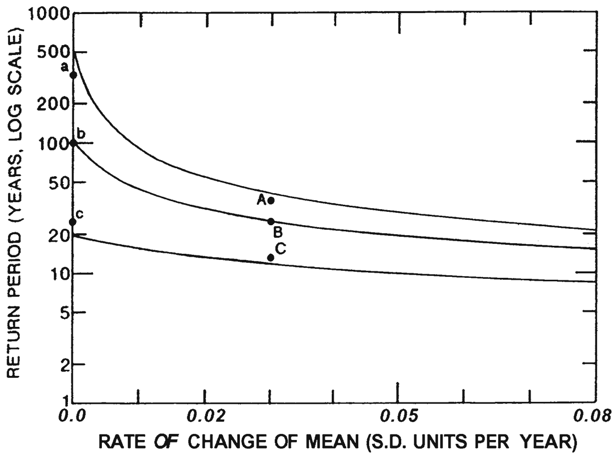

Fig. 4 As Fig. 2, but showing specific examples of June, July and August 1976. The return periods for these events in the absence of a trend in the mean are shown on the ordinate: a = June, b =July, c=August. Corresponding return periods for a representative warming rate of 0.03°C yr-1 (0.03 standard deviation units per year) are shown as points A, B and C

Table 1 The effect of a warming trend on the return periods (in years) for the June, July and August 1976 events

Month	Warming trend in degC yr^{-1} or S.D. units per year			
	0.00	0.01	0.03	0.05
June	330	75	36	26
July	100	44	25	19
August	25	18	13	11

associated with regional values, the real range of possible summer warming values must be somewhat larger than this.)

Now, the rate of warming depends not only on the equilibrium result, which is determined by the climate sensitivity, but also on the rate of future greenhouse forcing and on oceanic thermal inertia effects which cause a delay between the equilibrium and transient response. An equivalent $2 \times CO_2$ level (i.e. allowing for the effects of other trace gases such as methane, nitrous oxide and the CFCs) is expected to occur by around the year 2030. The corresponding transient warming is expected to lag substantially behind this. Just how long this lag is depends on the assumed forcing after 2030, and on the climate sensitivity, with longer lags for lower forcing and greater sensitivity. (This subject will be addressed at greater length in the next issue of Climate Monitor.)

The only practical way to tackling the lag problem is to consider the transient response directly using an appropriate time-dependent climate model. I have used the model of Wigley and Raper (1987) and considered the rates of change over 1990–2030 for different climate sensitivities and a range of possible future forcings. For the most likely forcing history, as given by Wigley and Raper but modified slightly to account for reduced CFC production under the Montreal Protocol (Wigley 1988), the rate of change of global-mean temperature over the period 1990–2030 varies from 0.016°C yr^{-1} for low climate sensitivity ($\Delta T_{2x} = 1.5°C$) to 0.037°C yr^{-1} high climate sensitivity ($\Delta T_{2x} = 4.5°C$).[1] If other possible future forcings are considered, the global-mean warming rate has an overall range of uncertainty of 0.01–0.05°C yr^{-1} (At the low end, the rate is comparable with rates observed this century. At the high end, the rate of warming is much faster than anything in the observational record.)

If these rates are, as suggested above, also representative of future summer temperature changes in England, then, since the σ values are about 1°C, the appropriate rates of change in σ-units are 0.01–0.05 yr^{-1}. The effects of such warming rates on the return period for the extreme warmth of 1976 are illustrated in Fig. 4 and summarized in Table 1. For an intermediate warming rate of 0.03 yr^{-1}, the return periods are: June, 36 years; July, 25 years; August, 13 years. Thus, for this warming rate, there is a high probability that the warmth of June 1976 will be equalled or surpassed at least once by the year 2030, and it is virtually certain that the warmth of July 1976 and August 1976 will be exceeded before 2030.

[1]For the latter case, the warming rate could well be somewhat faster than 0.037°C yr^{-1}. This is because there is a marked discrepancy between the observed global-mean warming to date and that which should have occurred. For observations to be compatible with a high climate sensitivity, some other internal or external forcing mechanism must be invoked to explain the anomaly. If this were removed, global-mean temperatures would have to warm at a faster rate than otherwise expected in order to "catch up" to the $\Delta T_{2x} = 4.5°C$ value.

Another way of looking at this is to note that the warm extremes that have occurred by chance over the past 300 years are likely to be compressed into the next 40 years, with new record highs almost guaranteed - and accelerated record-breaking as global warming continues into the twentyfirst century. Thus, if current greenhouse projections are valid, events which we now consider to be most unusual are likely to become commonplace well before the middle of next century.

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